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Output feedback controller design for input time-delayed nonlinear system

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Abstract

The linearization of an input/output controller has been designed for an input time delay nonlinear time discretized nonlinear system. The time discretized nonlinear model has been obtained based on Taylor-Lie series expansion method and zero order hold assumption. The resulting control algorithm enables the time delay nonlinear system control, while the continuous time controller cannot handle a time delay nonlinear system due to its infinite dimensionality. The performance of the proposed controller is evaluated by using two different case studies: a Van Der Pol equation and a Continuous Stirred Tank Reactor (CSTR) system that all exhibit nonlinear behavior and input time delay. For all the case studies, the results validate the proposed methods.

Keywords: Time discretized nonlinear systems; Time-delay systems; Digital control; Input/output linearization

1. Introduction

Time delay is often encountered in various engineering systems, such as chemical, hydraulic, and rolling mill systems and its existence is frequently a source of instability. Many of these systems are also significantly nonlinear which motivates research in control of nonlinear systems with time delay. Time-delay occurs at the gap from the beginning of an order to the response of the system due to a time interval, or spatial distance between the components of a control system [1]. The control systems with non-negligible time-delays exhibit complex behavior because of their infinite dimensionality. Even a linear time invariant system with a constant time delay in the input or state has infinite dimensionality, if expressed in continuous time domain. It is difficult to apply the controller design technique developed during the last several score years for finite-dimensional systems to the systems with any time-delays in the variables. Thus control system design methods for a discretized nonlinear system which can solve the system with time-delays are necessary.

Studies on time-delay have traditionally been conducted in the fields of chemical process control; however, the importance of the study for time-delay has been recently recognized by introducing a remote control system using networks [2-4]. In these cases, it is impossible to design a controller without considering the time-delay that inevitably occurs due to the spatial distance in signal transmission routes and network congestion.

Time-delay reduces gain and phase margins in a continuous system, causes a lowering of system performance, and makes the system unstable [1, 5]. A typical control method for a system that has time-delay is predictive control [6]. A control method using the Smith predictor has been proposed in the field of process control. This method has the merit that a controller can be designed by using a structural method, regardless of the effects of time-delay; how-

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ever, it can only be applied to a linear system. In addition, this method has the disadvantage that an exact model equation for the system and time-delay is required [1, 2, 5, 7].

An estimator is also proposed as an alternative method of predictive control. This estimator calculates state changes in the delayed time using an analysis of the time region of a state equation, and obtains an undelayed and exact plant state for the time that is required to calculate control signals. However, it is impossible to compensate for the time-delay for the input of the controller [4, 8].

Various techniques have been reported to resolve the time delay in nonlinear systems. Time Delay Control(TDC) is a control method that voluntarily introduces a small time delay in the control design, so as to reduce the effect of additive disturbances representing unknown dynamics[9-11]. Other techniques such as passivity control [12] and feedback linearization method [13] have been reported in the nonlinear time delay system controls. All these methods are for the continuous domain controllers.

Studies on time-delay have been largely conducted in a continuous time region. However, at the present time the controllers are designed by using a digital computer system. Thus, an important step is an analysis performed in the discrete-time region for a nonlinear system that has time-delay, and the design of a controller according to the analysis. Accordingly, this paper deals with designing a controller through a system discretization using a Taylor-series and linearization for inputs and outputs for a nonlinear system that has input time-delay.

This paper proposes a linearization of an input/output controller that has been designed for an input time delay nonlinear time discretized nonlinear system. The time discretized nonlinear system for input delayed was based on the Taylor-Lie series expansion and zero order hold assumption [14-16].

The discretization of the nonlinear system with input time-delay will be considered in Section 2. The controller design is discussed in Section 3 and in Section 4 case studies have been conducted. Finally, Section 6 concludes this research and describes the future work.

2. Discretization of a nonlinear system

A discrete-time model for a nonlinear continuous-time control system that has time-delay can be obtained by using a Taylor-series under the assumption of zero-order hold [14-16]. This discretization method provides a relatively exact discrete model, and makes it possible to apply the existing nonlinear control method to a discrete system, which includes time-delay.

A continuous-time nonlinear control system, which has a single input, can be presented as Eq. (1) using a state-space expression.

$$\frac{dx(t)}{dt} = f(x(t)) + g(x(t))u(t-D) \tag{1}$$

where $x \in X \subset R^n$ represents the system state, $u \in R$ is an input variable, D is time-delay, and $f(x)$ and $g(x)$ are nonlinear functions for x. In addition, a zero-order hold was assumed for a fixed sampling period, and constant input in a single sampling region.

$$\begin{aligned} &u(t) = u(kT) \equiv u(k) = \text{constant}, \quad kT \le t < kT + T \\ &D = qT + \gamma \end{aligned} \tag{2}$$

where T is sampling period, q is an integer multiple of $q \in \{1, 2, 3, \ldots\}$ for the sampling period, γ is a small time-delay of $0 < \gamma \le T$. The delayed input variable as shown in Fig. 1 was applied to the system that has values for the different sampling regions, as presented in Eq. (3).

$$u(t-D) = \begin{cases} u(k-q-1) & \text{if } kT \le t < kT + \gamma \\ u(k-q) & \text{if } kT + \gamma \le t < kT + T \end{cases} \tag{3}$$

A discrete system for a nonlinear system that has input time-delay can be configured as Eq.(4).

$$\begin{aligned} x(k+1) = {} & x(k) + \sum_{l=1}^{M} A^l(x(k), u(k-q-1)) \frac{\gamma^l}{l!} \\ & + \sum_{l=1}^{M} A^l((x(k) + \sum_{l=1}^{M} A^i(x(k), u(k-q-1)) \frac{r^i}{i!}), u(k-q)) \frac{(T-r)^l}{l!} \end{aligned} \tag{4}$$

where $x(k)$ is the value of a state vector of x at

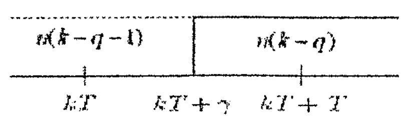

Fig. 1. Delayed input signal.

$t = t_k = kT$, M is truncation order of the Taylor-series. $A^{[l]}(x,u)$ is recursively defined by Eq. (5).

$$\begin{aligned} A^{[1]}(x, u) &= f(x)+ug(x) \\ A^{[l+1]}(x,u) &= \frac{\partial A^{[l]}(x, u)}{\partial x}(f(x)+ug(x)). \end{aligned} \tag{5}$$

The discrete expression for Eq. (1), can be presented by Eq. (6).

$$x(k+1) = \Phi_T^M (x(k), u(k-q-1), u(k-q)) \tag{6}$$

where the function Φ_T^M depends on the sampling period of T and truncation order of M .

3. Design of a controller

3.1 Linearization of input/output for a continuous-time nonlinear system

The input output linearization method linearizes a nonlinear system through an exact state transition and feedback, instead of a liner approximation method. Then, it applies a linear control method. This method transfers the original system into a simple equivalent model, and can be applied to control many industrial systems, such as aircraft, robotics, medical, and other various fields.

Let's consider the design of a controller for a nonlinear system.

$$\dot{x} = \Phi[x, u] \tag{7}$$

$$y = h[x] \tag{8}$$

where the output y and input u of the system can be obtained by using differentiation. The differential equation is expressed as a relative order of r . If the output y is infinite, the input will not affect the output. The order of output y of a control system should be finite and have relative order r for the input u .

$$\frac{d^l}{dt^l} y = h^{(l)}[x], \quad l = 0,\dots,r-1 \tag{9}$$

$$\frac{d^r}{dt^r} y = h^{(r-1)}[\Phi(x, u)] = f_2(x)u + f_1(x) \tag{10}$$

Eq. (10) can be obtained through differentiations

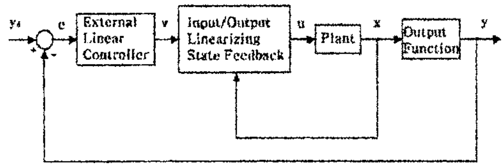

Fig. 2. Configuration of the design for input/output linearization.

with relative orders in order to verify the relationship between inputs and outputs. f_1 and f_2 are the functions for a system state. In Eq. (10), if the input u is configured by Eq. (11), the nonlinearity of Eq. (10) will be removed. Then, a simple linear differential equation for the output y and new internal input v can be obtained, as presented in Eq. (12).

$$u = \frac{1}{f_2}(v - f_1) \tag{11}$$

$$y^{(r)} = v \tag{12}$$

Since a linear control method can be applied to Eq. (12), a tracking control problem can be solved by using this method. When the internal input v can be configured as Eq. (14) using Eq. (12) and the differentiation of Eq. (13), which presents the output y and target value y_d , with relative orders, in this case the tracking error for the entire closed loop system is presented as Eq. (15)

$$e = y - y_d \tag{13}$$

$$v = y_d^{(r)} - k_r e^{(r-1)} - k_{r-1} e^{(r-2)} - \cdots - k_1 e \tag{14}$$

$$e^{(r)} + k_r e^{(r-1)} + k_{r-1} e^{(r-2)} + \cdots + k_1 e = 0 \tag{15}$$

If the coefficient k in each term of the equations is configured to satisfy a stable dynamics equation, in which Eq. (15) converges to zero, the input u that makes it possible to obtain a tracking property will be performed by applying a reverse process from Eq. (11) to Eq. (15).

Fig. 2 presents the configuration of design for input/output linearization controller of a nonlinear system.

3.2 Input/output linearization of a discrete-time nonlinear system

Design of a controller using input/output linearization for a nonlinear system in the discrete-time follows the same method in a continuous-time nonlinear system. Using the discrete-time nonlinear system presented in Eq. (16), the relative order and relative relationship between input and output can be verified as expressed in Eq. (17).

$$x(k+1)=\Phi[x(k),\, u(k)],\quad y(k)=h[x(k)] \tag{16}$$

$$y(k+r)=f_2(x(k))u(k)+f_1(x(k)) \tag{17}$$

where f_1 and f_2 are the functions related to the state of a discrete-time system. If the control input u is configured by Eq. (18), the nonlinearity of Eq. (17) will be removed. In addition, the system can be expressed as a simple linear differential equation for the output y and internal input ν, as presented in Eq. (19).

$$u(k)=\frac{1}{f_2(x(k))}\{\nu(k)-f_1(x(k))\} \tag{18}$$

$$y(k+r)=\nu(k) \tag{19}$$

As described above, the tracking problem for a linearized system is to be solved by using a linear control method. The internal input ν can be obtained as expressed in Eq. (21) using a transferring calculation with relative orders for Eq. (20). Then, the coefficient k is adjusted to satisfy a convergence property, which is required for a dynamics equation for the tracking error of the entire closed loop system presented in Eq. (22).

$$e(k)=y(k)-y_d(k) \tag{20}$$

$$\nu(k)=y_d(k+r)-k_r e(k+r-1) -k_{r-1}e(k+r-2)-\cdots-k_1 e(k) \tag{21}$$

$$e(k+r)+k_r e(k+r-1)+k_{r-1}e(k+r-2) +\cdots+k_1 e(k)=0 \tag{22}$$

In the tracking control issue of a discrete-time nonlinear system, the input u can be obtained by using reverse substitution of the mentioned processes. In addition, the system output converges to the target value of y_d by applying this value.

3.3 Design of a controller for a discrete-time nonlinear system with time-delay

The supplementary variables for the past input variables defined as Eq. (23).

$$\begin{aligned} z_1(k)&=u(k-q-1)\\ z_2(k)&=u(k-q)\\ &\vdots\\ z_{\hat{q}}(k)&=u(k-2)\\ z_{q+1}(k)&=u(k-1), \end{aligned} \tag{23}$$

Then, the dynamics equations are configured by Eq. (24).

$$\begin{aligned} z_1(k+1)&=z_2(k)\\ z_2(k+1)&=z_3(k)\\ &\vdots\\ z_q(k+1)&=z_{q-1}(k)\\ z_{q+1}(k+1)&=u(k) \end{aligned} \tag{24}$$

Thus, Eq. (1) can be expressed as a discrete-time nonlinear system as presented in Eq. (25), in which the equation presents an expanded state space.

$$\begin{bmatrix} x(k+1)\\ z_1(k+1)\\ \vdots\\ z_{q+1}(k+1) \end{bmatrix} = \begin{bmatrix} \Phi_T^M(x(k),\, z_1(k),\, z_2(k))\\ z_2(k)\\ \vdots\\ u(k) \end{bmatrix} \tag{25}$$

Let's define, $\bar{x}=[x, z_1, \cdots, z_{q+1}]^T$ and $\bar{\Phi}_T^M(\bar{x},u)=\begin{bmatrix} \Phi_T^M(x, z_1, z_2)\\ z_2(k)\\ \vdots\\ u \end{bmatrix}$, then Eq. (25) can be expressed as a simple formula, as presented in Eq. (26).

$$\bar{x}(k+1)=\bar{\Phi}_T^M(\bar{x}(k),\, u(k)) \tag{26}$$

The relative order of the system for the output equation of Eq. (27) can be derived by using a transferring operation, as presented in Eq. (28). In addition, if the control input is configured as Eq. (29), the nonlinearity of the system will be numerically removed.

$$y(k)=h(\bar{x}(k)) \tag{27}$$

$$y(k+r)=h^{r-1}\left(\bar{\Phi}_T^M(\bar{x}(k), u(k))\right) =F(k)+G(k)u(k)=\nu(k) \tag{28}$$

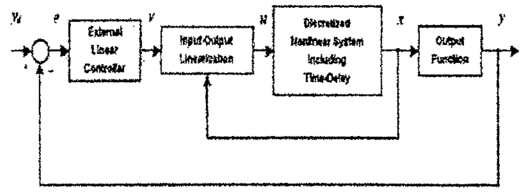

Fig. 3. Design of a controller for a nonlinear system with input time-delay.

$$u(k)=\frac{1}{G(k)}\{v(k)-F(k)\} \tag{29}$$

$F(k)=F\{\bar{x}(k)\}$ and $G(k)=G\{\bar{x}(k)\}$ in Eq. (29) are the function for the present state value and past input variables of $z(k)=[u(k-q-1),u(k-q),\cdots,u(k-1)]$, and $v(k)$ is the internal input.

In regards to the tracking issue, if the dynamics equation and internal input are defined by Eq. (30) and Eq. (31), the output property of the system is only followed by an error dynamics equation.

$$\begin{aligned} &e(k+r)+k_r e(k+r-1)+k_{r-1}e(k+r-2) \\ &\quad +\cdots+k_1 e(k)=0 \end{aligned} \tag{30}$$

$$\begin{aligned} v(k)=&(1+k_1+k_2+\cdots k_r)y_d-k_r y(k+r-1) \\ &-k_{r-1}y(k+r-2)-\cdots-k_1 y(k) \end{aligned} \tag{31}$$

Fig. 3 presents the configuration of the proposed control system.

4. Simulation

In order to verify the proposed method, we performed two simulations for nonlinear systems, with an input time-delay. The systems used in this simulation were a simple CSTR system and a Van der Pol equation. The Van der Pol system is a typical nonlinear system. It can be analyzed by using a mass-spring-damper system, which has a position-dependent damping coefficient, and an RLC electric circuit. If this system has an initial value besides an equilibrium point, a periodic vibration will be maintained in a limited region. This periodic vibration is called a limit cycle. Fig. 4 presents a phase portrait of the system.

The system can be expressed by a dynamics equation as presented in Eq. (32). The state space expression is Eq. (33).

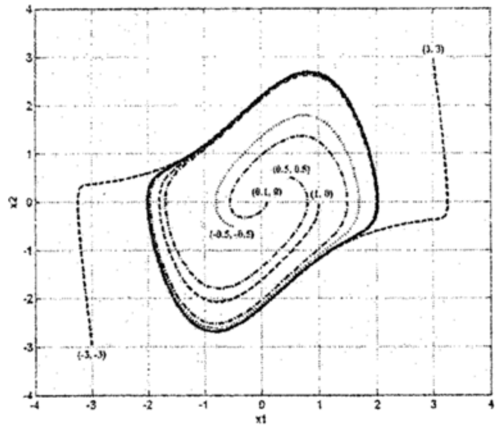

Fig. 4. Phase portraits of the Van der Pol system.

$$\begin{aligned} &\ddot{x}=\dot{x}(1-x^2)-x+u \\ &y=x, \end{aligned} \tag{32}$$

$$\begin{aligned} &\dot{X}_1=f_1(X)+g_1(X)u=X_2 \\ &\dot{X}_2=f_2(X)+g_2(X)u=X_2(1-X_1^2)-X_1+u. \end{aligned} \tag{33}$$

where the state vector is $X=[X_1\ X_2]=[x\ \dot{x}]^T$. In the case of the existing input time-delay, such as $D=qT+\gamma$, the discrete expression is expressed as Eq. (34) by using the Taylor's discretization method.

$$\begin{aligned} X_1(k+1)=&X_1(k)+\sum_{l=1}^{M}X_l\sum_{l=1}^{M}A_1^l(X(k),u(k-q-1))\frac{\gamma^l}{l!} \\ &+\sum_{l=1}^{M}A_1^l(X(k)+\sum_{l=1}^{M}A_1^j(x(k),u(k-q-1))\frac{\gamma^j}{r!}), \\ &u(k-q))\frac{(T-\gamma)^l}{l!} \\ X_2(k+1)=&X_2(k)+\sum_{l=1}^{M}A_2^l(X(k),u(k-q-1))\frac{\gamma}{l!} \\ &+\sum_{l=1}^{M}A_2^l((X(k)+\sum_{j=1}^{M}A_2^j(X(k),u(k-q-1))\frac{\gamma^j}{l!}), \\ &u(k-q))\frac{(T-\gamma)^l}{l!} \end{aligned} \tag{34}$$

where the time of the partial differentiation of $A^l(x,u)$ is to be cyclically defined as follows.

$$\begin{aligned} &A_1^l(x,u)=f_1(X)+g_1(X)u \\ &A_1^{l+1}(X,u)=\frac{\partial A_1^l(X,u)}{\partial X_1}f_1+\frac{\partial A_1^l(X,u)}{\partial X_2}f_2 \\ &A_2^l(X,u)=f_2(X)+g_2(X)u \\ &A_2^{l+1}(X,u)=\frac{\partial A_2^l(X,u)}{\partial X_1}f_1+\frac{\partial A_2^l(X,u)}{\partial X_2}f_2 \end{aligned} \tag{35}$$

Fig. 5 presents the output errors of a continuous

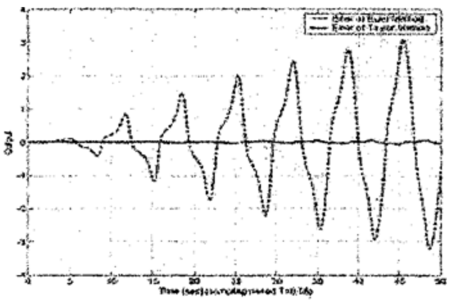

Fig. 5. Output error of the system with an initial value.

system and discrete system for the behaviors of a limited phase, in which the system has an initial value besides an equilibrium point. In addition, there is no input time-delay. This reveals that a discrete system using a Taylor-series presents superior characteristics to the existing Euler equation when applied to a discrete system.

$$x_{(0,0)} = [0.1 \quad 0]^T .$$

A controller was designed by using the input/output linearization method previously mentioned above for the input time-delay, such as *0T*, *1T*, *2T* and *3T*. The discretization was performed by using the truncation order of *M=2*. The relative orders for each case were obtained as Eq. (36). In addition, the output error dynamics equation was defined as Eq. (37), and the controller was designed as Eq. (38).

$$\begin{aligned}
y(k+1) &= F\{X_1(k), X_2(k)\} + (T^2/2!)u(k) \\
&= v(k) \\
y(k+2) &= F\{X_1(k), X_2(k), u(k-1)\} \\
&\quad + (T^2/2!)u(k) = v(k) \qquad (36) \\
y(k+3) &= F\{X_1(k), X_2(k), u(k-2), u(k-1)\} \\
&\quad + (T^2/2!)u(k) = v(k) \\
y(k+4) &= F\{X_1(k), X_2(k), u(k-3), \\
&\quad u(k-2), u(k-1)\} \\
&\quad + (T^2/2!)u(k) = v(k)
\end{aligned}$$

$$\begin{aligned}
& e(k+1) + k_1 e(k) = 0, \quad k_1 = -0.7 \\
& e(k+2) + k_2 e(k+1) + k_1 e(k) = 0, \quad k_2 = -1.4, k_1 = 0.53 \\
& e(k+3) + k_3 e(k+2) + k_2 e(k+1) + k_1 e(k) = 0 \\
& k_3 = -0.7, k_2 = -0.45, k_1 = 0.371 \\
& e(k+4) + k_4 e(k+3) + k_3 e(k+2) \\
& \quad + k_2 e(k+1) + k_1 e(k) = 0
\end{aligned}$$

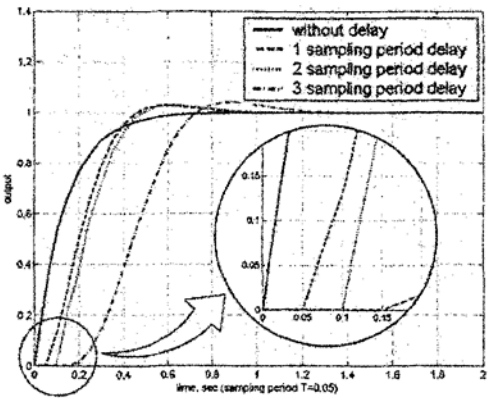

Fig. 6. Output of the nonlinear control system with input time-delay.

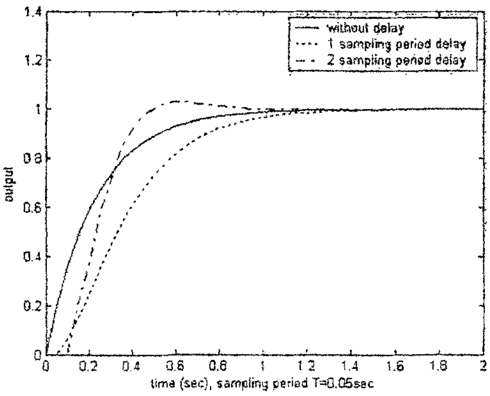

Fig. 7. Output of the CSTR system with input time-delay.

$$k_4 = -2.8, k_3 = 3.02, k_2 = -1.484, k_1 = 0.2809 \qquad (37)$$

$$u(k) = \frac{1}{(T^2/2!)}\{v(k) - F(k)\} \qquad (38)$$

Fig. 6 presents the output of the system for each case where the output shows a simple time-transition removed by the time-delay effect.

A second computer simulation was conducted for the CSTR system. A CSTR system is a common nonlinear system in a chemical process. The dynamic equation of the system is presented in Eq. (39).

$$\begin{aligned} x' &= -x^2 - 3x + (1-x)u \\ y &= x \end{aligned} \qquad (39)$$

In the case of the existing time-delay, such as $D = qT + \gamma$, the discrete expression is expressed as Eq. (4) and Eq. (5) by using the Taylor's discretization method. A controller was designed with the input/output linearization method previously mentioned above for the input time-delay, such as *0T*, *1T* and *2T*. The discretization was performed by using a truncation order of $M=2$.

Fig. 7 presents the output of the system for each case where the output shows a simple time-transition removed by the time-delay effect as the first simulation result.

5. conclusions

In order to compensate for the input time-delay of a nonlinear system, this study proposed a design method that supports an exact discretization for a nonlinear system that has time-delay, and designed a controller for a discrete system that includes time-delay using a Taylor-series. The proposed control system that has the characteristics of an independent output for time-delay was verified by simulation. Although with this method the system has to be re-analyzed according to the time-delay, it has the merit that the system can be applied to a large time-delay and variable time-delay.

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